

REVIEW & SUMMARY

Position Vector The location of a particle relative to the origin of a coordinate system is given by a *position vector* \vec{r} , which in unit-vector notation is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}. \quad (4-1)$$

Here $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$ are the vector components of position vector \vec{r} , and x , y , and z are its scalar components (as well as the coordinates of the particle). A position vector is described either by a magnitude and one or two angles for orientation, or by its vector or scalar components.

Displacement If a particle moves so that its position vector changes from \vec{r}_1 to \vec{r}_2 , the particle's *displacement* $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \quad (4-2)$$

The displacement can also be written as

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \quad (4-3)$$

$$= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}. \quad (4-4)$$

Average Velocity and Instantaneous Velocity If a particle undergoes a displacement $\Delta\vec{r}$ in time interval Δt , its *average velocity* \vec{v}_{avg} for that time interval is

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}. \quad (4-8)$$

As Δt in Eq. 4-8 is shrunk to 0, \vec{v}_{avg} reaches a limit called either the *velocity* or the *instantaneous velocity* \vec{v} :

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad (4-10)$$

which can be rewritten in unit-vector notation as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}, \quad (4-11)$$

where $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$. The instantaneous velocity \vec{v} of a particle is always directed along the tangent to the particle's path at the particle's position.

Average Acceleration and Instantaneous Acceleration If a particle's velocity changes from \vec{v}_1 to \vec{v}_2 in time interval Δt , its *average acceleration* during Δt is

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}. \quad (4-15)$$

As Δt in Eq. 4-15 is shrunk to 0, \vec{a}_{avg} reaches a limiting value called either the *acceleration* or the *instantaneous acceleration* \vec{a} :

$$\vec{a} = \frac{d\vec{v}}{dt}. \quad (4-16)$$

In unit-vector notation,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \quad (4-17)$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.

Projectile Motion *Projectile motion* is the motion of a particle that is launched with an initial velocity \vec{v}_0 . During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$. (Upward is taken to be a positive direction.) If \vec{v}_0 is expressed as a magnitude (the speed v_0) and an angle θ_0 (measured from the horizontal), the particle's equations of motion along the horizontal x axis and vertical y axis are

$$x - x_0 = (v_0 \cos \theta_0)t, \quad (4-21)$$

$$y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2, \quad (4-22)$$

$$v_y = v_0 \sin \theta_0 - gt, \quad (4-23)$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0). \quad (4-24)$$

The **trajectory** (path) of a particle in projectile motion is parabolic and is given by

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}, \quad (4-25)$$

if x_0 and y_0 of Eqs. 4-21 to 4-24 are zero. The particle's **horizontal range** R , which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g} \sin 2\theta_0. \quad (4-26)$$

Uniform Circular Motion If a particle travels along a circle or circular arc of radius r at constant speed v , it is said to be in *uniform circular motion* and has an acceleration \vec{a} of constant magnitude

$$a = \frac{v^2}{r}. \quad (4-34)$$

The direction of \vec{a} is toward the center of the circle or circular arc, and \vec{a} is said to be *centripetal*. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v}. \quad (4-35)$$

T is called the *period of revolution*, or simply the *period*, of the motion.

Relative Motion When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B . The two measured velocities are related by

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}, \quad (4-44)$$

where \vec{v}_{BA} is the velocity of B with respect to A . Both observers measure the same acceleration for the particle:

$$\vec{a}_{PA} = \vec{a}_{PB}. \quad (4-45)$$