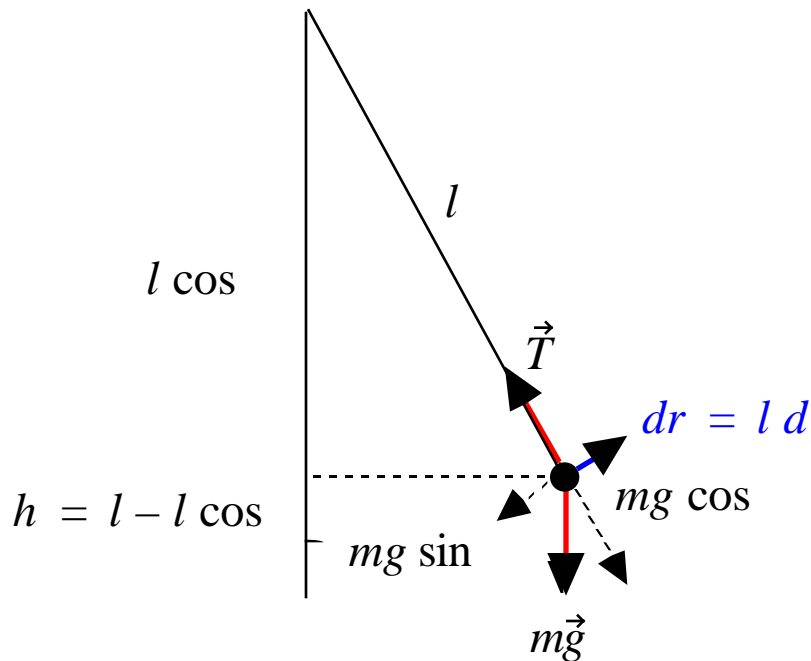


Work done by the Earth on a pendulum



$$\begin{aligned}
 W &= \int \vec{F} \cdot d\vec{r} \\
 &= - \int_i^f mgl \sin \theta \, d\theta \\
 &= mgl \cos \theta_f - mgl \cos \theta_i \\
 &= mgh_i - mgh_f
 \end{aligned}$$

since the gravitational pull of the earth is the only force doing work on the pendulum. Tension does no work since $\vec{T} \cdot d\vec{r} = 0$

$$K = W$$

$$\begin{aligned}
 \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 &= mgh_i - mgh_f \\
 \frac{1}{2}mv_f^2 + mgh_f &= \frac{1}{2}mv_i^2 + mgh_i
 \end{aligned}$$

Total mechanical energy is conserved, since $U = mgh$.